*Eur. J. Mech. B/Fluids*


# IS THE SUPPRESSION OF SHORT WAVES BY A SWELL A THREE-DIMENSIONAL EFFECT?

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(Received 16 September 1998, revised and accepted 28 December 1998)

**Abstract** – We consider the phenomenon of suppression of short waves by a long wave, observed by Mitsuyasu in 1966. The recently proposed [1] essentially 3-D explanation of this phenomenon is reviewed and compared with more traditional 2-D explanations. Several physical implications of this 3-D explanation are suggested and the experimental verification is discussed. 

## 1. Introduction

More than thirty years ago H. Mitsuyasu [2] noticed the following quite strange phenomenon, that still puzzles scientists. He performed experiments with a large water tank ($70m$ long, $8m$ wide, and $3m$ deep). A blower, suspended above the water surface, could produce a wind wave field on the water surface. A wave paddle located at one side of the tank could generate a longer wave (the opposite side of the tank had an absorbent beach in order not to deal with reflected waves). The wind-generated waves were about 4-5 times shorter than the mechanically generated waves. Mitsuyasu compared the energy spectrum of the surface wave field in two situations: (1) when only the blower produced waves (the paddle was at rest); and (2) when the wave field was produced by the blower and by the paddle working simultaneously. He found that under the same wind conditions (i.e. same wind from the blower) the energy of the short-wave field was noticeably smaller in the case of the working wave paddle than in the case of the resting one. This appears quite strange: the working wave paddle puts some extra energy into the system, and we could expect a larger excitation of the system. Instead, the short wave field calms down.

In 1974 Phillips and Banner [3] suggested a possible explanation of this phenomenon. It is well known that a wind blowing for some time generates the so called drift current (under the water surface). Phillips and Banner argued that the presence of the drift current and the long wave causes short waves to break at lower amplitudes, and therefore the short wave field turns out to be smaller in the presence of the long wave.

Phillips and Banner considered the 2-D situation (1-D water surface), and the subsequent studies of this phenomenon were concentrated on the 2-D case. The experiments were often performed in narrow water tanks, whose width did not exceed $1m$ while their length exceeded $10m$ (see e.g. Kusaba and Mitsuyasu [4], Yuen [5], and Cheng and Mitsuyasu [6]). The theoretical developments in the understanding of this phenomenon were also often based on the 2-D models (Caponi and Saffman [7] and Morland [8]).

Recently Balk [1] suggested another possible explanation of this phenomenon, which is essentially three-dimensional; the proposed mechanism of the suppression can work only in 3-D situations (2-D water surface). Though the phenomenon may look like two-dimensional, it actually could possess some hidden three-dimensionality.

The goal of the present note is to point out at some physical implications of the 3-D mechanism and at some experiments that could distinguish whether the phenomenon of the suppression is due to the 2-D or 3-D effects. In Section 2 we review the 3-D mechanism and in Section 3 formulate its physical implications.

## 2. The mechanism

In order to obtain a quantitative model, we assume that the waves have sufficiently small amplitudes, so that we can take advantage of the wave kinetic equation

$$\begin{aligned}\frac{\partial n_1}{\partial t} &= \int R_{1234}\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)\delta(\omega_1+\omega_2-\omega_3-\omega_4)\times \\ &\times[n_1n_3n_4+n_2n_3n_4-n_1n_2n_3-n_1n_2n_4]d\mathbf{k}_2d\mathbf{k}_3d\mathbf{k}_4.\end{aligned} \tag{1}$$

Here $\omega_{\mathbf{k}}$ is the dispersion law; for the deep-water gravity waves $\omega(\mathbf{k})=\sqrt{k}$ $(k=|\mathbf{k}|)$; $n_{\mathbf{k}}=\varepsilon_{\mathbf{k}}/\omega(\mathbf{k})$ is the wave action spectrum ($\varepsilon_{\mathbf{k}}$ is the energy spectrum). We use the following notational contractions: $\omega_i=\omega(\mathbf{k}_i)$, $n_i=n_{\mathbf{k}_i}$ $(i=1,2,3,4)$; $R_{1234}=R(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$. The kernel $R$ is positive and is determined by the form of the medium's nonlinearity.

We assume (as was in the experimental setting) that the amplitudes of the short waves are much smaller than the amplitude of the long wave, characterized by some wave vector $\mathbf{p}_0$. More precisely, the wave action spectrum $n_{\mathbf{k}}$ has a sufficiently sharp peak at $\mathbf{k}=\mathbf{p}_0$, so that the main contribution to the "collision integral" (1) comes from the region where two of the integration vectors $\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4$ are "close" to the vector $\mathbf{p}_0$. Then the kinetic equation (1) can be reduced to a differential equation. This calculation is presented in [1]; here we only note that when $\mathbf{k}_2$ and $\mathbf{k}_4$ are close to $\mathbf{p}_0$, the frequency $\delta$-function in (1) becomes

$$\begin{aligned}\delta[\omega(\mathbf{k}_1)+\omega(\mathbf{k}_2)-\omega(\mathbf{k}_3)-\omega(\mathbf{k}_4)] &= \delta[\omega(\mathbf{k}_1)-\omega(\mathbf{k}_3)+\frac{\partial\omega}{\partial\mathbf{k}}(\mathbf{p}_0)\cdot(\mathbf{k}_2-\mathbf{k}_4)]= \\ &= \delta[\omega(\mathbf{k}_1)-\omega(\mathbf{k}_3)-\frac{\partial\omega}{\partial\mathbf{k}}(\mathbf{p}_0)\cdot(\mathbf{k}_1-\mathbf{k}_3)]=\delta[\Omega(\mathbf{k}_1)-\Omega(\mathbf{k}_3)]\end{aligned}$$

where

$$\Omega(\mathbf{k})=\omega(\mathbf{k})-\frac{\partial\omega}{\partial\mathbf{p}}(\mathbf{p}_0)\cdot\mathbf{k} \tag{2}$$

is the dispersion law of the short waves in the frame of reference moving with the group velocity of the long wave. Hence the wave action spectrum $n_{\mathbf{k}}$ of the short-wave field evolves along the level lines of the function (2)

$$\Omega(\mathbf{k})=C=\text{const.} \tag{3}$$

In other words, the time derivative of the spectrum $n_{\mathbf{k}}$ on each curve (3) depends only on the values of the spectrum $n_{\mathbf{k}}$ on the same curve; it is independent of the spectrum $n_{\mathbf{k}}$ on other curves (3). In the case of gravity waves, the curves (3) are shown in *figure 1.*

Thus the intensive long wave causes the wave action of the short wave field to redistribute along the curves (3), i.e. roughly speaking, in the direction orthogonal to the direction of propagation of the long wave. Here the three-dimensionality is crucial.

The differential equation, to which the wave kinetic equation is reduced, is actually an equation of one-dimensional diffusion. To write down this equation, we transfer from the Cartesian coordinates $\mathbf{k}=(\xi,\eta)$ to the variables $\Omega=\Omega(\xi,\eta)$, $\eta$:

$$\frac{\partial n(\Omega,\eta,t)}{\partial t}=\left|\frac{\partial\Omega}{\partial\xi}\right|\frac{\partial}{\partial\eta}\left[D(\Omega,\eta)\frac{\partial n}{\partial\eta}\right]; \tag{4}$$

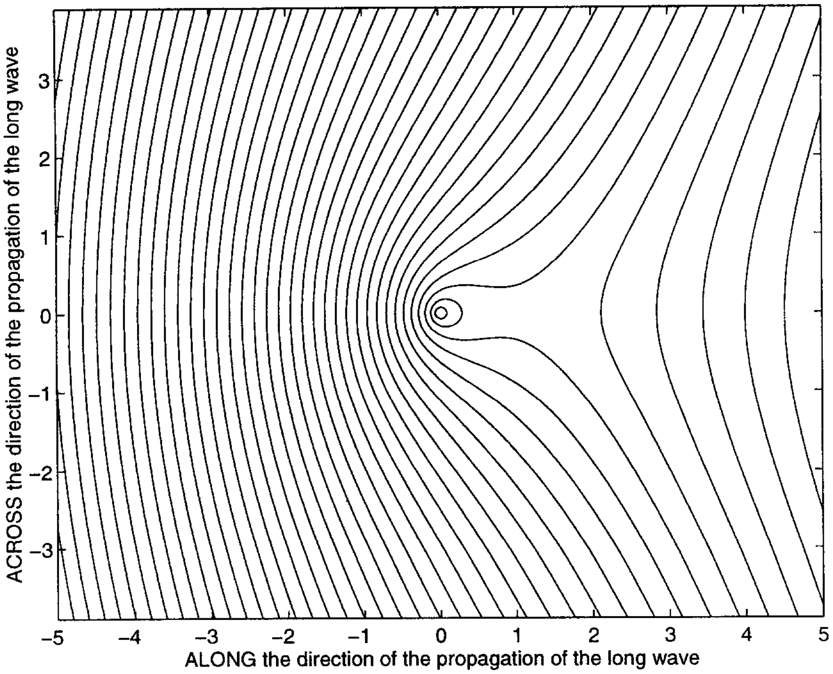


FIGURE 1. In the case of deep-water gravity waves $\Omega(\mathbf{k}) = (\xi^2+\eta^2)^{\frac{1}{4}} - \frac{1}{2\sqrt{p_0}}\xi$ where $\xi,\eta$ are components of the wave vector $\mathbf{k}$ respectively along and across the direction of propagation of the long wave. The diffusion of $n_{\mathbf{k}}$ occurs in the $\mathbf{k}=(\xi,\eta)$-plane along these curves $\Omega(\mathbf{k}) = \text{const}$ (the units are chosen such that $p_0 = 1$).

here the derivatives with respect to $\eta$ are calculated for a fixed $\Omega$. The diffusion coefficient $D(\Omega,\eta)$ is given by the following formula

$$D(\Omega,\eta) = R(\mathbf{k},\mathbf{p}_0,\mathbf{k},\mathbf{p}_0)\left|\frac{\partial\Omega}{\partial\xi}\right|^{-2}\int u^2 n_{\mathbf{p}+\mathbf{q}/2}n_{\mathbf{p}-\mathbf{q}/2}d\mathbf{p}du, \quad \text{where} \quad \mathbf{q} = \left(-u\frac{\partial\Omega/\partial\eta}{\partial\Omega/\partial\xi},\ u\right).$$

The diffusion leads to the equipartition of the wave action on each of the curves (3). In the absence of dissipation, the total wave action on each of the curves (3)

$$\mathcal{N}(C) = \int n_{\mathbf{k}}\delta(\Omega(\mathbf{k})-C)d\mathbf{k} \tag{5}$$

remains constant. Since the curves (3) are not closed and go to infinity, the distribution $n_{\mathbf{k}}$ vanishes with time. One may say that the long wave is sweeping the short waves to the region of large $|\mathbf{k}|$ where they dissipate, and thereby, the long wave suppresses the short-wave field.

## 3. Physical Implications

In real situations the wave kinetic equation could be not applicable (since the kinetic description of the interaction between the short wave field and the train of long waves might require an extremely small nonlinearity level). However, the mechanism of suppression remains qualitatively the same: the long wave causes redistribution of the wave action of the short-wave field in the direction transversal to the direction of the propagation of the long wave. The curves may "smear" out, and the redistribution may take place in the longitudinal direction as well, but at a slower rate than in the transversal direction. Herewith the simple diffusion may be replaced by some complicated relaxation process.

Thus the 3-D mechanism could be rejected or confirmed by the experiments in *wide* water tanks (like the one in the original Mitsuyasu's experiment [2]). One should measure the 2-D spectrum of the surface height (in longitudinal and transversal directions). According to the 3-D mechanism (see Section 2), the turning on the wave paddle (the presence of an intensive long wave) should lead to the spreading of the spectrum of the short wave field in the transversal direction. The short wave field almost one-dimensional in the beginning would become essentially two-dimensional.

We should note that the proposed mechanism can take place only if the long wave is sufficiently intensive, such that the dynamics of short waves is mainly due to the (nonlocal) interaction with the long wave train, while other interactions can be neglected. For example, the local cascade interactions could actually lead to the intensification of the short wave field due to the flux of energy from the long wave to the short waves. Which interaction is dominant is determined by the actual form of the spectrum. Paper [1] gives some estimates.

It is interesting that the long wave, actually, supplies energy to short waves, but this gain of energy makes them dissipate faster. Indeed, on each of the curves (3), the total wave action (5) is conserved and is transported to the region of higher wave numbers $k$, and therefore, to higher energies $\omega(\mathbf{k})$ and higher dissipation. One can say, using the language of social analogy, that the long wave promotes short waves to the positions with higher salaries (higher energies), but this makes them die sooner (dissipate faster).

Since the long wave gives some of its energy to the short waves, the amplitude of the long wave should attenuate faster if it passes through the short-wave field. This effect was indeed observed by Mitsuyasu [2].

It would be also informative to perform experiments in wide (3-D) water tanks with various angles between the wind from the blower and the long wave generated by the paddle. According to the proposed 3-D mechanism the effect should be qualitatively independent of this angle in 3-D tanks (as well as in field observations).

The 3-D mechanism has a general physical nature. It is not related to a drift current and shows that the phenomenon of suppression of short waves by a long wave can be observed in experiments with different physical systems.

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